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# Variable Equation of State Parameter for Accelerating Expansion of the Universe

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***Abstract:*** **Equation of state parameter depending on energy density is derived using a gauge – like condition. It shows the possibility of expressing the accelerating expansion with positive energy. when $-1 \leq \omega, \widetilde{\omega} \leq 1$, It laid out energy density constrains as $\rho \geq \frac{1}{2} A\rho_0$, $\widetilde{\rho} \geq \frac{1}{2} A\widetilde{\rho}_0$ a gauge – like condition has been used in this work, is a result of multiplication of special constructed octonion. Octonion that used in this model verified as mixed octonion, constructed out of two quaternions, one represents the field and the other is its dual field.**

***Keywords:*** **Equation of state parameter, accelerating expansion, universe, dark energy.**

## I. INTRODUCTION

Accelerating expansion of the Universe became literarily in the 1990s while studying the explosion of supernova stars [1]. The confirmation for acceleration arisen from observations of supernovae, Type Ia supernovae [2]. This accelerating expansion is thought to be caused by dark energy, which pertains all of space and increase the rate of expansion of the Universe, because of its negative equation of state parameter[3-4].

Somehow, the dynamics of dark energy properties are treated as perfect fluid, obeying the equation of state of perfect fluid with negative parameter of the equation [5].

The role of equation of state parameter $\omega$ in studying the evolution of the universe, proposed it as an identifier of the consequence phases of the universe that passed since the big bang.

However, time dependent equation of state parameter $\omega(z)$, defined as the ratio of its pressure to its energy density, became relevance in studying the contribution of dark energy in accelerating expansion of the universe, after the observational results coming from SN Ia data [6] and SN Ia data collaborated with CMBR anisotropy and galaxy clustering statistics [7].

Vázquez et al, investigate the evolution of time dependent equation of state parameter with redshift by performing a Bayesian analysis of cosmological observations [8].

In this work we introduced an equation of state frame by using a gauge – like condition. This condition is a consequence of multiplication of special constructed octonion, which we called it mixed octonion, it is a combination of field and its dual field. This gauge – like condition construes to that of Lorenz gauge when the scalars part of the octonion vanished.

## II. MIXED OCTONION

As far as we know Octonion is Hyper Holomorphic space $\mathbb{R}^8$, it can be defined as a set of two quaternions [9]. Similarly as Hamilton called vector quaternions right quaternions [10,11] and real numbers (considered as quaternions with zero vector part) scalar quaternions, we construct octonion from right quaternion contains the field and left quaternion contains its dual field, we call it mixed octonion, defines as

$$\tilde{A} = a_0 + \acute{a}_0 + \vec{A} + \overleftarrow{A}\ ;\ \tilde{B} = b_0 + \acute{b}_0 + \vec{B} + \overleftarrow{B} \qquad (1)$$

Where the field F and its dual field $F_D$ are define with

$$F = f_0 + \vec{F} \quad ; \quad F_D = \acute{f}_0 + \overleftarrow{F} \qquad (2)$$

Fields and their duals are defining as

$$\vec{A} = a_1\hat{e}_1 + a_2\hat{e}_2 + a_3\hat{e}_3 \quad , \overleftarrow{A} = \underline{e}^1\acute{a}_1 + \underline{e}^2\acute{a}_2 + \underline{e}^3\acute{a}_3 \tag{3}$$

$$\vec{B} = b_1\hat{e}_1 + b_2\hat{e}_2 + b_3\hat{e}_3 \quad , \overleftarrow{B} = \underline{e}^1\acute{b}_1 + \underline{e}^2\acute{b}_2 + \underline{e}^3\acute{b}_3 \tag{4}$$

The left vector $\overleftarrow{A}$ is the dual vector of right vector $\vec{A}$, $a_0$ , $b_0$, $\acute{a}_0$ and $\acute{b}_0$ are scalars ( either number or operators, real or imaginary).

Multiplication of two mixed octonions $\widetilde{A}$ and $\widetilde{B}$ can be done as a combination of two quaternions as

$$\tilde{A}\tilde{B} = (a_0 + \acute{a}_0)\cdot(b_0 + \acute{b}_0) - \{\vec{A}\cdot\vec{B} + \overleftarrow{A}\cdot\overleftarrow{B}\} - \{\vec{A}\cdot\overleftarrow{B} + \overleftarrow{A}\cdot\vec{B}\} + \{a_0\vec{B} + \acute{a}_0\vec{B} + \vec{A}b_0 + \vec{A}\acute{b}_0\} + \{a_0\overleftarrow{B} + \acute{a}_0\overleftarrow{B} + \overleftarrow{A}b_0 + \overleftarrow{A}\acute{b}_0\} + \{\vec{A}\times\overleftarrow{B} + \overleftarrow{A}\times\vec{B}\} + \vec{A}\times\vec{B} + \overleftarrow{A}\times\overleftarrow{B} \tag{5}$$

Either the dual field or the field vanished, the multiplication in (5) construes to quaternion multiplication.

If equation (5) splitting with condition $\widetilde{A}\widetilde{B} = 0$ we find

$$\{\vec{A}\times\overleftarrow{B}\} + \{\overleftarrow{A}\times\vec{B}\} = 0 \tag{6}$$

Equation (6) reveals the conservation due to total (field+dual field) space.

The crossing multiplication can be shown as

$$\overleftarrow{A}\vec{B} + \vec{A}\overleftarrow{B} = 0 \tag{7}$$

$$a_0(b_0 + \acute{b}_0) + \acute{a}_0(b_0 + \acute{b}_0) - \{\vec{A}\cdot\vec{B} + \overleftarrow{A}\cdot\overleftarrow{B}\} = 0 \tag{8}$$

Equation (8) reveals the relation between dot product of field and its dual field. It can be written as

$$\overleftarrow{A}\cdot\overleftarrow{B} = (a_0 + \acute{a}_0)\cdot(b_0 + \acute{b}_0) - \vec{A}\cdot\vec{B} \tag{9}$$

Equation (9) expresses a gauge – like condition. This gauge condition reduces to that of Lorenz when the scalars equal zero. That, what Arbab was proposed in Propagation of matter wave in a medium [12].

**Equation of state:**

If we define $\tilde{V} = (\frac{c\rho}{\rho_0}, -\frac{c\tilde{\rho}}{\tilde{\rho_0}}, \overleftarrow{v}, \vec{v})$ and $\tilde{\nabla} = (\frac{1}{c}\vec{v}\cdot\vec{\nabla}, -\frac{\overleftarrow{v}}{c}\cdot\overleftarrow{\nabla}, \overleftarrow{\nabla}, \vec{\nabla})$ , where ρ is the energy density of the field and $\tilde{\rho}$ is the energy density of its dual field.

We find velocity divergence from equation (9) as

$$\vec{\nabla}\cdot\vec{v} + \overleftarrow{\nabla}\cdot\overleftarrow{v} = \vec{v}\cdot\vec{\nabla}\Omega - \overleftarrow{v}\cdot\overleftarrow{\nabla}\Omega \tag{10}$$

Where the density parameter $\Omega = \frac{\rho}{\rho_0} - \frac{\tilde{\rho}}{\tilde{\rho_0}}$

Equation (10) describes a fluid. If the fluid velocity vector field is divergence-free

$(\vec{\nabla}\cdot\vec{v} + \overleftarrow{\nabla}\cdot\overleftarrow{v}) = 0$ , equation (10) reads

$$\vec{\nabla}\cdot(\Omega\vec{v}) + \frac{\partial\Omega}{\partial t} = 0 \tag{11}$$

Where $\overleftarrow{v}\cdot\overleftarrow{\nabla} = -\frac{\partial}{\partial t}$ and we use the identity $\vec{\nabla}\cdot(f\vec{F}) = \vec{F}\cdot\vec{\nabla}f + f\vec{\nabla}\cdot\vec{F}$ with equation (10) and condition $\vec{\nabla}\cdot\vec{v} = 0$ .

Equation (11) is continuity equation for fluid. It is interesting to find the density parameter satisfies the continuity equation.

If the fluid is an incompressible flow (ρ is constant), the mass continuity equation simplifies to a volume continuity equation [13], and this leads to

$$\Omega(\rho, \tilde{\rho}) = \frac{\rho}{\rho_0} + \frac{\tilde{\rho}}{\tilde{\rho_0}} = A \tag{12}$$

Where A is constant.

Equation (12) construes to equation of state

$$\grave{p} = \omega\rho \qquad (13)$$

By defining the pressure as the energy density of the dual fluid $\tilde{p}(\rho) \equiv \tilde{\rho}$ , we can calculate equation of state parameter as

$$\omega(\rho) = \frac{\widetilde{\rho_0}}{\rho_0}\left(\frac{A\rho_0}{\rho} - 1\right) \qquad (14)$$

Hence, equation (13) becomes

$$\tilde{p}(\rho) = \frac{\widetilde{\rho_0}}{\rho_0}\left(\frac{A\rho_0}{\rho} - 1\right)\rho \qquad (15)$$

On the other hand when $p(\tilde{\rho}) \equiv \rho$

$$\widetilde{\omega}(\tilde{\rho}) = \frac{\rho_0}{\widetilde{\rho_0}}\left(\frac{A\widetilde{\rho_0}}{\tilde{\rho}} - 1\right) \qquad (16)$$

Equations (14,16) reveal positive, negative and zero values, depending on positive values of energy densities.

Plugging equation (16) in equation (13) one finds

$$p(\tilde{\rho}) = \frac{\rho_0}{\widetilde{\rho_0}}\left(\frac{A\widetilde{\rho_0}}{\tilde{\rho}} - 1\right)\tilde{\rho} \qquad (17)$$

Both equation (15) and equation (17) are equivalent in describing one kind of field. Therefore, total equation of state that describes the composed system (field + dual field) is

$$P = \frac{\widetilde{\rho_0}}{2\rho_0}\left(\frac{A\rho_0}{\rho} - 1\right)\rho + \frac{\rho_0}{2\widetilde{\rho_0}}\left(\frac{A\widetilde{\rho_0}}{\tilde{\rho}} - 1\right)\tilde{\rho} \qquad (18)$$

Where $P = \frac{\tilde{p}(\rho)+p(\tilde{\rho})}{2}$.

The singularities in equations (14) and equation (16) prevent vanishing either field or the dual field energy to zero.

If we assume $\frac{\rho_0}{\rho} \gg 1$ , equation (12) implies $\frac{\widetilde{\rho_0}}{\tilde{\rho}} \gg 1$. Hence, equation (18) gives

$$P = \frac{1}{2}(\rho_0 + \widetilde{\rho_0})A \qquad (19)$$

Equation (19) clarifies the mechanism of expansion in the universe that occupied with both field and its dual field. If $\rho$ represents the field (e.g. baryonic matter, radiation) and $\tilde{\rho}$ represents the dual field (e.g. dark matter) $\frac{\rho_0}{\rho} \gg 1$ , shows at large scale of the universe, the energy density decreasing. The pressure became constant, depending on the initial energy densities of the field and the dual field.

If we reverse the conditions, $\frac{\rho_0}{\rho} \ll 1$ and $\frac{\widetilde{\rho_0}}{\tilde{\rho}} \ll 1$ we find

$$P = -\left(\frac{\widetilde{\rho_0}}{2\rho_0}\rho + \frac{\rho_0}{2\widetilde{\rho_0}}\tilde{\rho}\right) \qquad (20)$$

Equation (20) also permits expansion reasons by the field energy at the beginning of the universe evolution, where the energy density is very large so $\frac{\rho_0}{\rho} \ll 1$ and $\frac{\widetilde{\rho_0}}{\tilde{\rho}} \ll 1$ .

Inflationary phase at the very earliest age of the universe expansion would be driven by energy, which emerges from the decay of a particle field and produces a universe many times larger than a simple linear expansion [14]. Inflation predicts the origin for the growth of large-scale structure in the universe. Since the universe would be inflated several orders of magnitudes, it would essentially have flat space geometry.

Hence, we have two regimes at the early time of the universe age and the late time of the universe age, where the universe accelerates and inflates.

Finally we conclude with calculating the values of energy density, equation (14) and equation (16) in certain cases of universe evolution as shown in table (1).

**TABLE I: shows various values of energy density during the universe evolution with $\rho_0 = \tilde{\rho}_0$**

| $\omega, \tilde{\omega}$ | $1$ | $\frac{1}{3}$ | $0$ | $-\frac{1}{3}$ | $-1$ |
|---|---|---|---|---|---|
| $\rho$ | $\frac{1}{2}A\rho_0$ | $\frac{3}{4}A\rho_0$ | $A\rho_0$ | $\frac{3}{2}A\rho_0$ | $\infty$ |
| $\tilde{\rho}$ | $\frac{1}{2}A\tilde{\rho}_0$ | $\frac{3}{4}A\tilde{\rho}_0$ | $A\tilde{\rho}_0$ | $\frac{3}{2}A\tilde{\rho}_0$ | $\infty$ |

As we can see equation of state parameter cannot exceeds $-1$. This limit known as the Phantom Divide Line (PDL)[15]. Moreover, if $-1 \le \omega, \tilde{\omega} \le 1$ , this imposes constrains and limits the variation of energy density to be $\rho \ge \frac{1}{2}A\rho_0$ and $\tilde{\rho} \ge \frac{1}{2}A\tilde{\rho}_0$

## III. CONCLUSION

The dependence of equation of state parameter on energy densities as in equation (14) and equation (16), gives rise to describe dark energy with positive energy rather than negative one. Equation (18) is effective equation of state that can be used in studying cosmic inflation and the accelerated expansion of the universe.

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